Supplementary Material

Biases in regional carbon budgets from covariation of surface fluxes and weather in transport model inversions

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S1. Inverse transform sampling method

The stochastic model for surface and horizontal CO$_2$ fluxes ($F$), entrainment ($E$), and boundary layer height ($h$) accounts for the cross-covariances (or cross-spectra in the frequency domain) of each time-series at time-scales from daily to seasonal (by solving equations 4-7). The model does not account for non-Gaussian distributions of these variables, including skewness and extreme values. We therefore transformed the Gaussian outputs of the stochastic model back to the original distributions of the inputs by fitting a cumulative density function to the original data using kernel density estimation. Each Gaussian output time-series was then transformed to a uniform distribution (using the probability integral transform), then back to the original distribution by applying the inverse of the estimated cumulative density function. The inverse transform acts on the probability distributions, and preserves the cross-covariances of the modeled time-series.

As an example of how well the transformed stochastic model output captures the characteristics of the original time-series, we show the original time-series of CT-TM5 inputs in Fig. (S1), and the stochastically-modeled time-series in Fig. (S2). The modeled time-series are shown for the first 8 years of the stochastic simulation (out of 300 years simulated). The average seasonal cycle has been added back to the synthetic time-series (after detrending and deseasonalizing) for the purpose of comparison to the original data. Skewness in the time-series distributions plays more of a role for entrainment and
boundary layer height, which are positive-definite, than surface and horizontal CO$_2$ fluxes.

Fig. S1. Original time-series from the CT-TM5 dataset for 2002-2010. Top panel: Combined surface fluxes and horizontal advection (F). Middle panel: Entrainment rate (E). Bottom panel: Boundary layer height (h).
Fig. S2. Synthetic CT-TM5 time-series for the first 8 years of the stochastic simulation. Top panel: Combined surface fluxes and horizontal advection (F). Middle panel: Entrainment rate (E). Bottom panel: Boundary layer height (h).